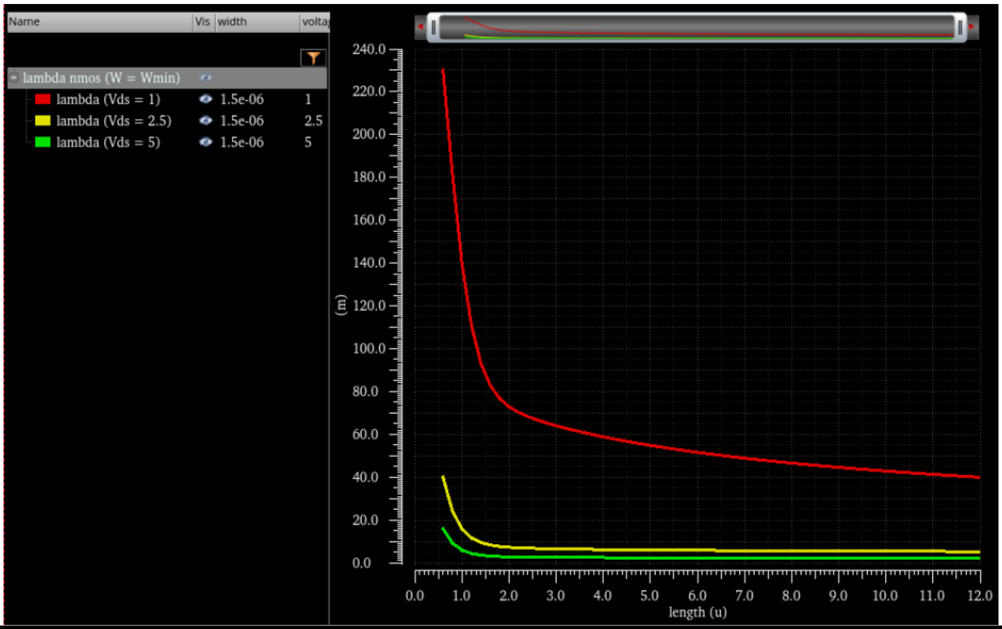
EE 435

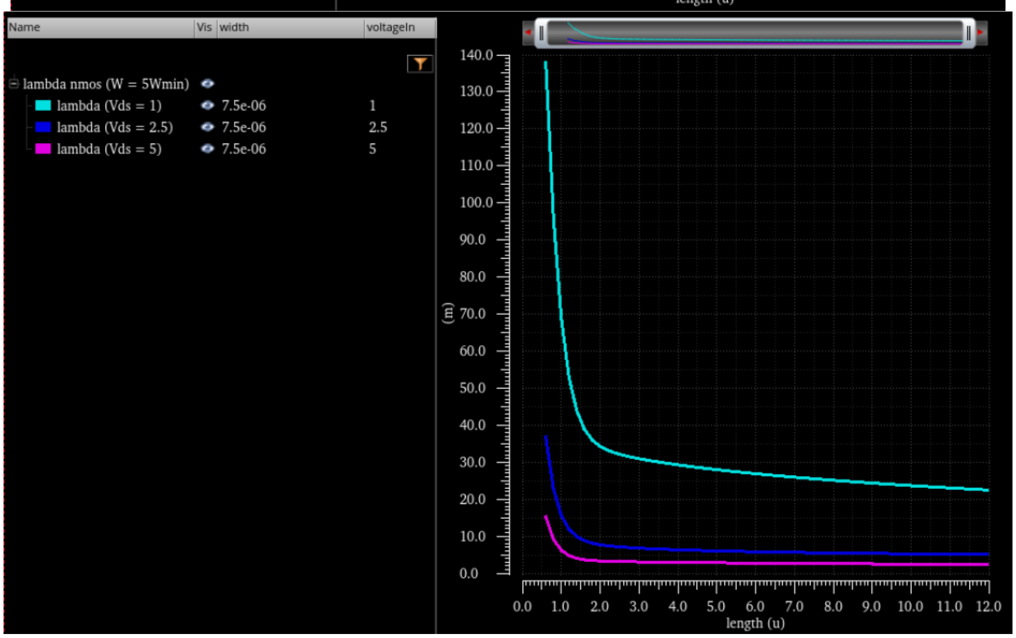
Homework 4 Solutions

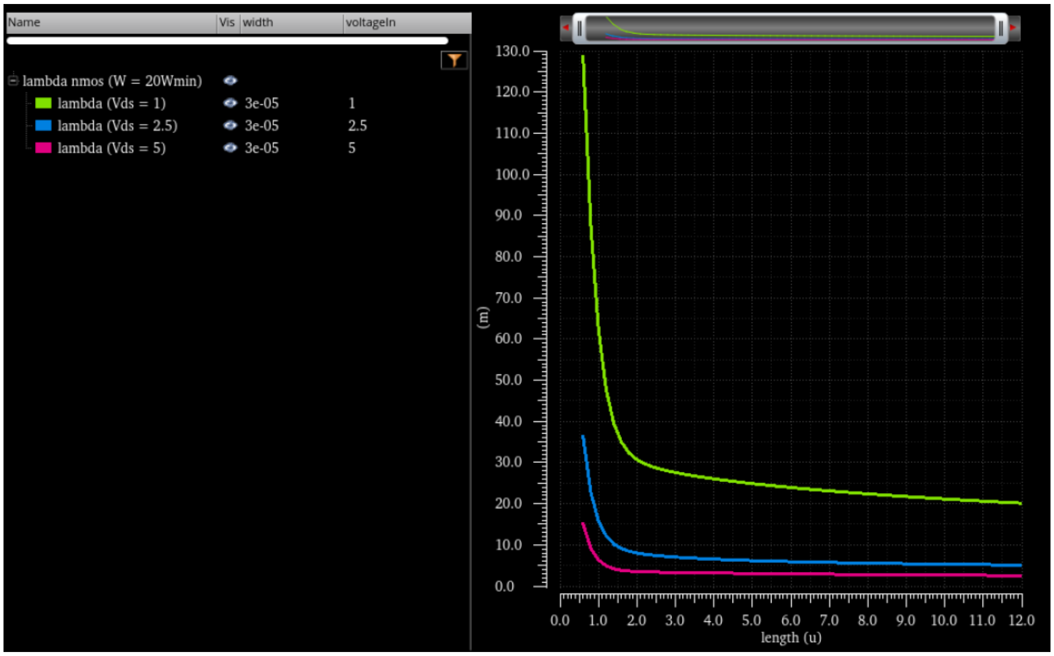
Spring 2024

Problem 1 & 2

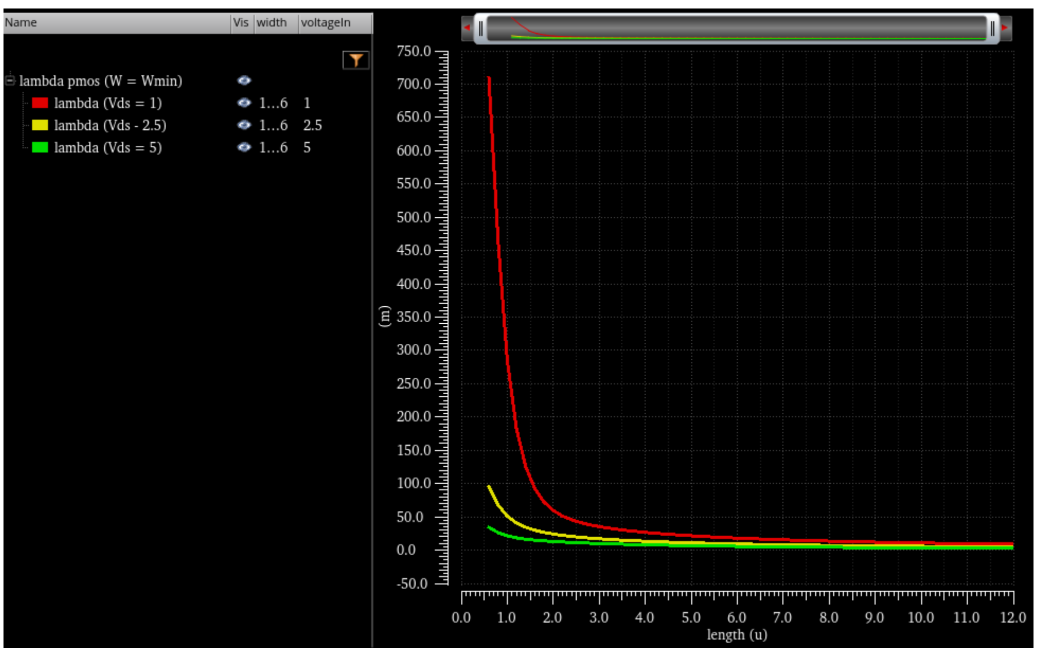
NMOS

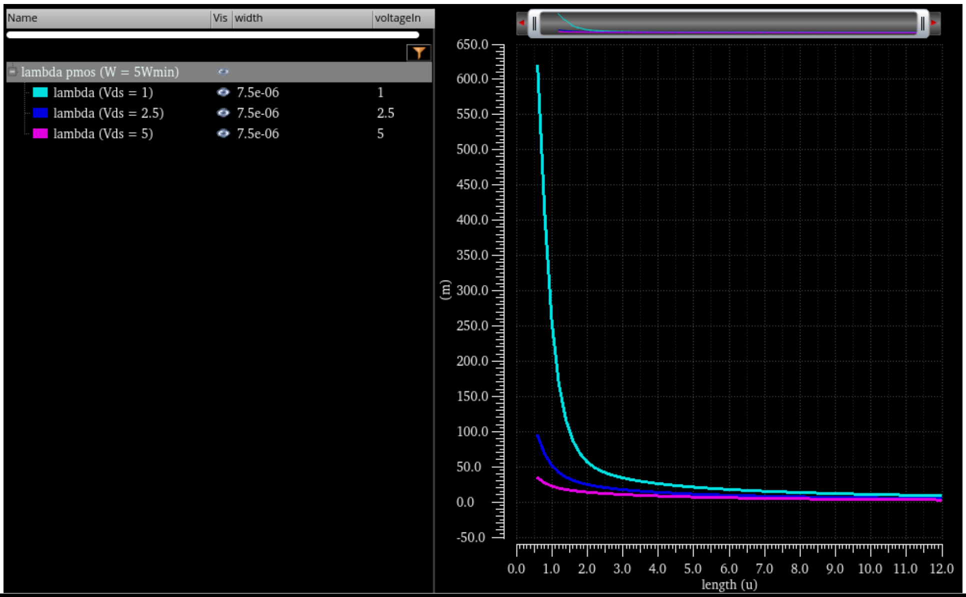


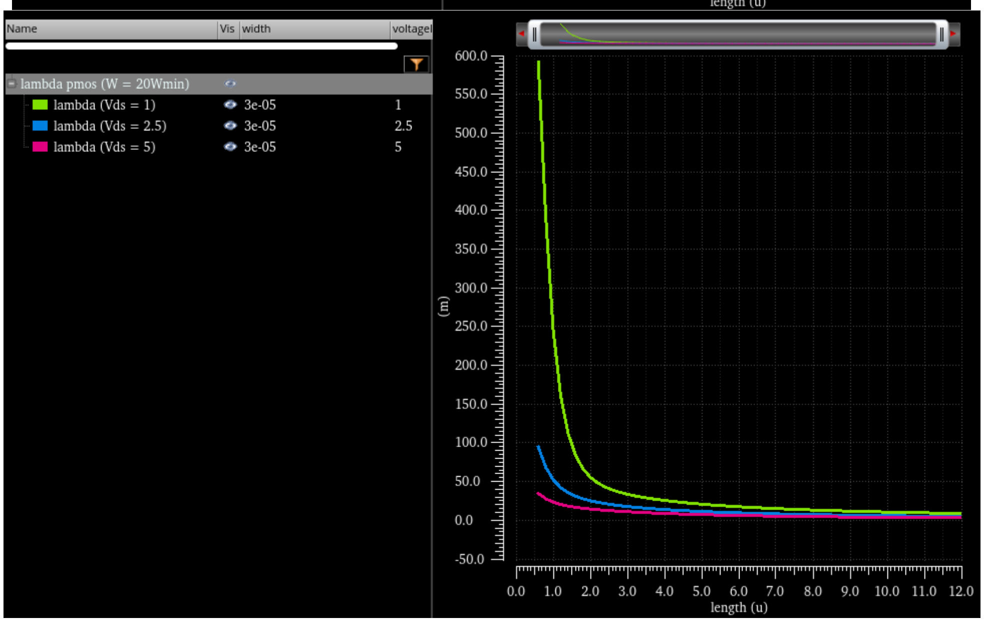



Name
Vis
width
voltageIn
lambda nmos (W = 20Wmin)
lambda (Vds = 1) 3e-05 1
lambda (Vds = 2.5) 3e-05 2.5
lambda (Vds = 5) 3e-05 5
(m)
length (u)

PMOS


Name
Vis
width
voltageIn
lambda pmos (W = Wmin)
lambda (Vds = 1) 1...6 1
lambda (Vds - 2.5) 1...6 2.5
lambda (Vds = 5) 1...6 5
(m)
length (u)





Problem 3

Part A

The current through M1 is $\frac{P}{V_{DD}-V_{SS}} * \frac{1}{2} = 83.33\mu A$. So we can find W_1 as foll

$$W_1 = \frac{2IL}{\mu C_{ox} V_{EB}^2} = \frac{2 * 83.3\mu * 2\mu}{112.8\mu * 0.2^2} = 73.84\mu m$$

Part B

We know that $V_{EB1} = V_{GS} - V_{Tn} = 0.2V$. Quiescently, $V_G = 0V$. So:

$$V_G - V_S - V_{Tn} = -V_S - 0.79V = 0.2 \rightarrow V_S = -0.99V$$

Part C

Create an expression for V_{OUT} divided by V_{IN}:

$$g_{m1}V_{IN} = V_{OUT}\left(sC_1 + \frac{1}{R_1}\right)$$
$$A(s) = \frac{g_{m1}}{sC_1 + 1/R_1}$$

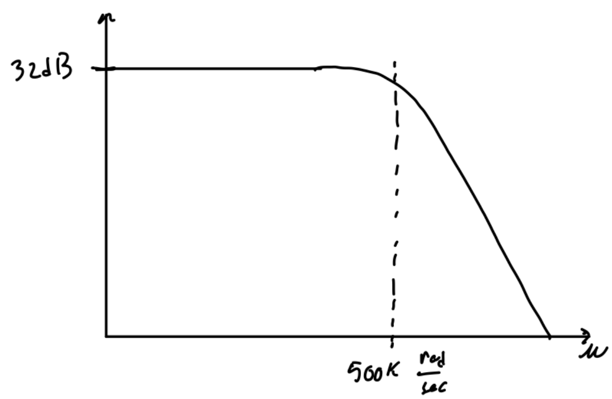
Part D

Start by finding the DC gain:

$$A_0 = g_m R_1 = \frac{2 * 83.33\mu A}{0.2} * 50k\Omega = 41.665_{10} = 32.39dB$$

Now find the corner frequency:

$$sC_1 + \frac{1}{R_0} = 0 \rightarrow s = -\frac{1}{R_1 C_1} = 500k\ rad/sec$$



Part E

As found in Part D, the 3dB bandwidth is $500k\ rad/sec$.

Part F

Increasing the power increases the gain-bandwidth, but not the bandwidth. The bandwidth is only dependent on R_1 and C_1.

Problem 4

Part A

Pole spread, k, is defined simply to be p_2/p_1. The open-loop amplifier has poles located at $p_2 = 10^4\ rad/sec$ and $p_1 = 5\ rad/sec$. The pole spread is then:

$$k = \frac{10^4}{5} = 2000$$

Part B

Begin by finding the amplifier's closed loop transfer function:

$$A_{CL}(s) = \frac{A(s)}{1+\beta A(s)} = \frac{\frac{10^9}{(s+5)(s+10^4)}}{1+\frac{\beta 10^9}{(s+5)(s+10^4)}} = \frac{10^9}{(s+5)(s+10^4)+10^9\beta}$$

$$= \frac{10^9}{s^2+s(10005)+50000+\beta 10^9}$$

Let $\beta = 0.1$:

$$A_{CL}(s) = \frac{10^9}{s^2+s(10005)+50000+10^8}$$

To find the pole Q, it is easiest to recall the following standard form:

$$D(s) = s^2 + \frac{p}{Q}s + p^2$$

We can use this to solve for Q:

$$p = \sqrt{50000+10^8}$$

$$\frac{p}{Q} = \frac{\sqrt{50000+10^8}}{Q} = 10005 \rightarrow Q = 0.99975$$

Part C

To find overshoot, ζ, start by finding the pole Q again with the new β.

$$A_{CL}(s) = \frac{10^9}{s^2+s(10005)+50000+5*10^8}$$

$$p = \sqrt{50000+5*10^8}$$

$$\frac{p}{Q} = \frac{\sqrt{50000+5*10^8}}{Q} = 10005 \rightarrow Q = 2.235$$

Now recall that $\zeta = \frac{1}{2Q}$.

$$\zeta = \frac{1}{2Q} = \frac{1}{4.47} = 0.22$$

Part D

Per Lecture 13, slide 12, to avoid ringing in the step response the pole Q should be $\frac{1}{2}$. We can then work backward to figure out what β gives this pole Q:

$$\frac{p}{Q} = \frac{\sqrt{50000 + \beta 10^9}}{1/2} = 10005 \to \beta = 0.0249$$

The maximum closed-loop DC gain is $\frac{1}{\beta} = 40$.

Part E

If we assume that the DC gain of the open-loop amplifier is not changed by adjusting the location of p_1 (as stated by the problem), then we can assume A_0 will continue being $20k\ V/V$. To achieve an optimal response with no ringing, we want a pole spread equal to $4\beta A_0$ (Lecture 13, Slide 12). In our case, $\beta = 1$ and $A_0 = 20kV/V$. So, we need a pole spread of $80k$.

If p_2 remains at $10^4\ rad/sec$, we can find p_1 easily:

$$k = 80k = \frac{p_2}{p_1} = \frac{10^4}{p_1} \to p_1 = \frac{10^4}{80k} = 0.125\ rad/sec$$

The minimal adjustment is then $5 - 0.125 = 4.875\ rad/sec$.

Problem 5

Part A

Begin by figuring out how much current flows through each branch of the amplifier. Let I_{BRANCH} be the current through the left- and right-most branches (M_5, M_6, M_8, and M_9) and I_{TAIL} b the current through the middle branch (M_7).

The total power consumption of the amplifier structure is given to be $10mW$. The net current is then $I_{NET} = \frac{10mW}{2V} = 5mA$.

Because M_{35} and M_{46} are both given to be 20, we can find the branch and tail current:

$$I_{TAIL} + 2I_{BRANCH} = 5mA$$
$$I_{TAIL} + 2\left(20 * \frac{I_{TAIL}}{2}\right) = 5mA$$
$$I_{TAIL} + 20I_{TAIL} = 5mA$$
$$I_{TAIL} = 238\mu A$$
$$I_{BRANCH} = \frac{20I_{TAIL}}{2} = 2.38mA$$

If I know the V_{EB} and drain current of each MOSFET, I can find the device size easily using the square-law equation for a MOSFET in saturation. Use the process parameters given at the top of the homework. That gives the below results:

Device	W/L
M_1 & M_2	105.77
M_3 & M_4	320.53
M_5 & M_6	6410.77
M_7	211.55
M_8 & M_9	2115.55

Part B

By inspection:

$$A_0 = -\frac{Mg_{m1}}{g_{o6} + g_{o8}}$$
$$GBW = \frac{g_{m1}}{C_L}$$

Recalling that $g_m = \frac{2I_{DQ}}{V_{EB}}$ and $g_o \approx \lambda I_{DQ}$:

$$g_{m1} = \frac{20 * 2\frac{238\mu A}{2}}{0.15} = 31.72m$$
$$g_{o6} = g_{08} \approx 0.01V^{-1}I_{BRANCH} = 23.8\mu\Omega^{-1}$$
$$A_0 = -\frac{31.72m}{47.6\mu} = -666 = 56.4dB$$
$$GBW = \frac{31.72m}{10pF} = 3.172G\frac{rad}{sec} = 504.83MHz$$

Part B

$$g_{MEQ} = M * g_{m1} = M * \frac{2I_{DQ}}{V_{EB}} = 20 * 2 * \frac{\frac{238\mu A}{2}}{0.15} = 31.73mS$$

Problem 6

Part A

Let the positive input to the g_{m2} OTA be V_x. Start by defining two equations for the currents at V_x and V_{OUT}:

$$(V_x - V_{IN})sC_1 + g_{m1}V_{OUT} = 0$$
$$V_{OUT}sC_2 - g_{m2}(V_x - V_{OUT}) = 0$$

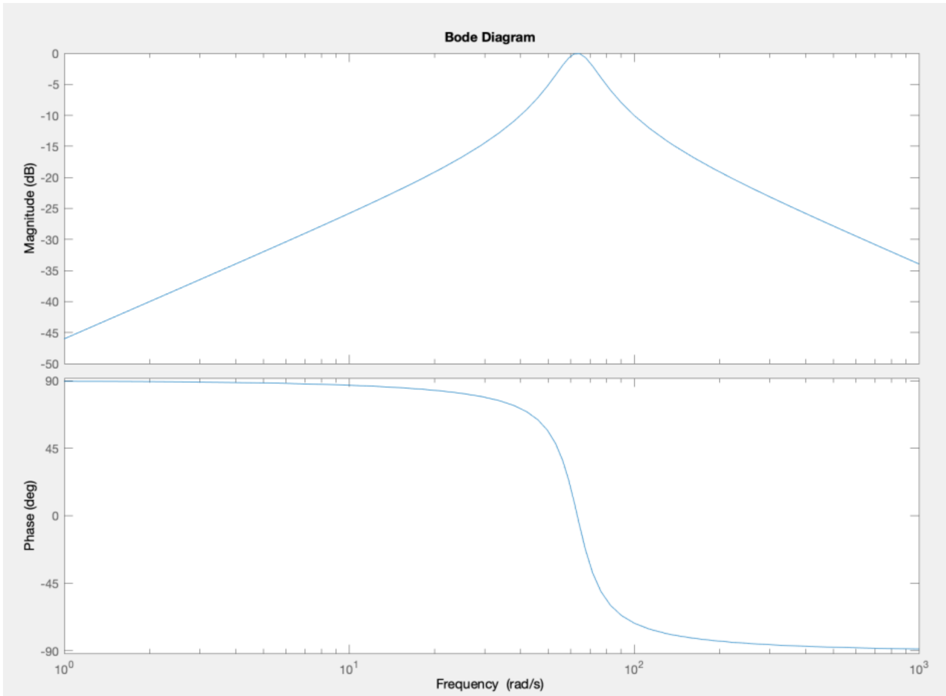
Solve the first equation for V_x:

$$V_xsC_1 - V_{IN}sC_1 - g_{m1}V_{OUT} = 0 \rightarrow V_x = \frac{V_{IN}sC_1 - g_{m1}V_{OUT}}{sC_1}$$

Substitute into the second equation:

$$V_{OUT}sC_2 - g_{m2}\left(\frac{V_{IN}sC_1 - g_{m1}V_{OUT}}{sC_1} - V_{OUT}\right) = 0$$
$$V_{OUT}\left(sC_2 + \frac{g_{m2}g_{m1}}{sC_1} + g_{m2}\right) = V_{IN}(g_{m2})$$
$$V_{OUT}\left(\frac{s^2C_1C_2 + g_{m2}g_{m1} + g_{m2}sC_1}{sC_1}\right) = g_{m2}V_{IN}$$
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{sC_1g_{m2}}{s^2C_1C_2 + g_{m2}sC_1 + g_{m2}g_{m1}}$$

Part B



Problem 7

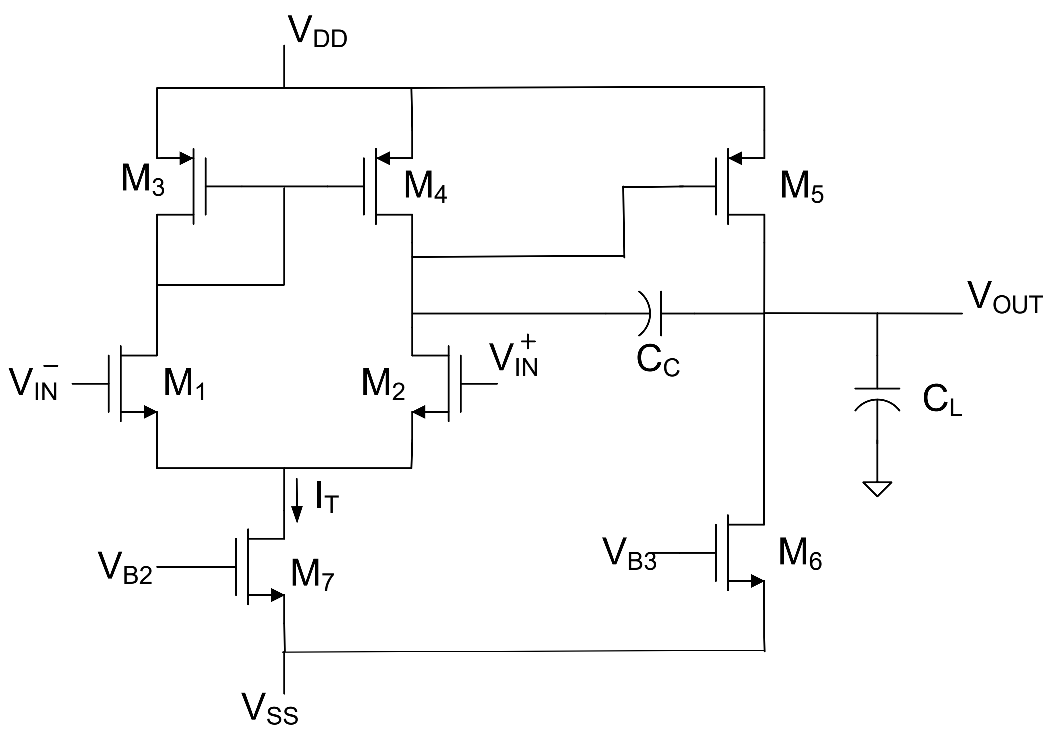
Consider the polynomial $D(s) = s^2+1500s+3000$. It was pointed out in the lecture that when a second-order polynomial has widely separated poles on the negative real axis that the high-frequency pole can be closely approximated by considering only the s^2 and s terms and the low frequency pole can be closely approximated by considering only the constant and the s-term in this expression. Compare the actual roots and the approximate roots for D(s) and comment on how much error is introduced by using this approximation approach.

Solution: From the approximation given in class, from $s^2+1500s$, we obtain $p_2=-1500$ and From $1500s+3000$ we obtain $p_1=-2$. From the quadratic equation, the two roots are actually: $p_2=-1498$ and $p_1=-2.0027$. It can be observed that the approximation is very good for these widely separated poles.

Problem 8 and 9

Consider the 7-T op amp where all transistors are sized for $V_{EB}=0.1V$ with a power dissipation of 2mW split evenly between the first and second stage and where Miller compensation is used. Assume this is designed in a 0.18μm CMOS process with $V_{DD}=1.2V$, $V_{SS}=-1.2V$, and $C_L=100fF$. Assume the process is characterized by parameters $\mu_nC_{OX}=300\mu AV^{-2}$, $V_{THn}=0.5V$, $V_{THp}=-0.5V$, $\mu_p=\mu_n/3$, and $\lambda=0.01V^{-1}$ for both the n-channel and p-channel devices.

a) Determine V_{B2} and V_{B3}
b) Determine W/L for all transistors
c) What is the dc gain of this operational amplifier?
d) Determine C_C if the feedback amplifier is to have a closed-loop pole Q of 0.707 when β=0.2.
e) What is g_{m5}?



Solution:

a) $V_{B2}-V_{SS}=V_{THn}+V_{EB}$ Thus $V_{B2}=V_{SS}+V_{THn}+V_{EB}$ = (-1.2+0.5+0.1)V = -0.6V. By the same argument, V_{B3}= -0.6V.

b) For all transistors, $I_D = \frac{\mu C_{OX} W}{2L} V_{EB}^2$. Thus $\frac{W}{L} = \frac{2I_D}{\mu C_{OX} V_{EB}^2}$. Since 1mW is dissipated in each stage and since the total supply voltage is 2.4V, it follows that $I_T=I_{D6}$=0.417mA and I_{D1}=0.209 mA. It thus follows that W1/L1=139, W3/L3=417, W7/L7=278, W5/L5=834 and W6/L6=278.

c) $A_0 = \frac{4}{(\lambda_n + \lambda_p)^2 V_{EB1} V_{EB5}} = \frac{4}{.02^2 0.1^2} = 10^6$

d)

$$C_C = \frac{C_L\, 2\theta(1-\theta)\beta}{Q^2} \frac{V_{EB1}|V_{EB5}|}{\left(V_{EB1} 2\theta - \beta |V_{EB5}|(1-\theta)\right)^2}$$

With θ=0.5, it follows that and C_L=100fF, it follows that C_C=24.7fF

e) $g_{m5}=2I_{DQ5}/V_{EB}$ I_{DQ5}*2.4V=1mW so I_{DQ5}=0.42mA
thus g_{m5}=8.4E-3 AV^{-1}